

Exhibit C

Fundamentals of Inertial Navigation, Satellite-based Positioning and their Integration

Department of Electrical and Computer
Engineering
Royal Military College of Canada/
Queen's University
Kingston
Canada

Trusted Positioning Inc.
Calgary
Canada

Tashfeen B. Karamat
Department of Electrical
and Computer Engineering
Queen's University
Kingston
Canada

ISBN 978-3-642-30465-1 ISBN 978-3-642-30466-8 (eBook)
DOI 10.1007/978-3-642-30466-8
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2012945733

© Springer-Verlag Berlin Heidelberg 2013

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publisher's location, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Figure 7.2 shows the discrete-time system corresponding to Eqs. (7.1) and (7.2).

The state transition matrix (STM) Φ represents the known dynamic behavior of the system (in this case the INS error model) which relates the state vector from epoch $k - 1$ to k . Given the dynamic coefficient matrix F of a continuous time system the STM is

$$\Phi = \exp(F\Delta t) \quad (7.3)$$

To linearize this for use by KF we take the first two terms of the Taylor series expansion of the equation as follows

$$\Phi = (I + F\Delta t) \quad (7.4)$$

where I is identity matrix and Δt is sampling interval.

7.1.1 KF Assumptions

Kalman filtering relies on the following assumptions (Maybeck 1979; Minkler and Minkler 1993).

1. The system (both the process and the measurements) can be described by linear models.
2. The system noise \mathbf{w}_k and the measurement noise $\boldsymbol{\eta}_k$ are uncorrelated zero-mean white noise processes with known auto covariance functions, hence

$$E[\mathbf{w}_k] = 0, \quad E[\boldsymbol{\eta}_k] = 0 \quad \forall k \quad (7.5)$$

$$E[\mathbf{w}_k \boldsymbol{\eta}_j^T] = 0 \quad \forall k, j \quad (7.6)$$

$$E[\mathbf{w}_k \mathbf{w}_j^T] = \begin{cases} Q_k, & k = j \\ 0 & k \neq j \end{cases} \quad (7.7)$$

$$E[\boldsymbol{\eta}_k \boldsymbol{\eta}_j^T] = \begin{cases} R_k, & k = j \\ 0 & k \neq j \end{cases} \quad (7.8)$$

where Q_k and R_k are known positive definitive matrices. In INS/GPS integration, Q_k represents the covariance matrix of the system noise associated with the INS errors, and R_k represents the covariance matrix of the measurement noise associated with the GPS position and velocity updates.

3. The initial system state vector \mathbf{x}_0 is a random vector uncorrelated to both the process and measurement noises, hence

$$E[\mathbf{x}_0 \mathbf{w}_k^T] = 0, \quad E[\mathbf{x}_0 \boldsymbol{\eta}_k^T] = 0 \quad \forall k \quad (7.9)$$